Semi-leptonic decays of heavy mesons

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We present results of a lattice computation of the matrix elements of the vector currents which are relevant for the semileptonic decays of $D \to K$, $D \to \pi$ and $B \to \pi$. The computations are performed in the quenched approximation on a $24^3 \times 48$ lattice at $\beta = 6.2$, using an $\mathcal{O}(a)$ non-perturbatively improved fermionic action.

1. INTRODUCTION

The matrix elements of heavy meson decays, particularly B mesons, are important for the determination of Cabibbo-Kobayashi-Maskawa matrix elements. We present preliminary results of a lattice study of vector current matrix elements for heavy-to-light semi-leptonic transitions.

2. SIMULATION DETAILS

We used the Wilson action to generate 216 $SU(3)$ gauge configurations on a $24^3 \times 48$ lattice at $\beta = 6.2$. The fermionic degrees of freedom were computed using an $\mathcal{O}(a)$ improved clover action with a non-perturbative value for C_{SW} [1]. The inverse lattice spacing, set by the ρ mass is $a^{-1} = 2.64$ Gev. Four values of the heavy quark hopping parameter were used: 0.1200, 0.1233, 0.1266, 0.1299, where 0.1233 roughly corresponds to the charm quark mass, and all 4 are used to extrapolate to the b quark mass region. Three values of the light quark hopping parameter were used: 0.1346, 0.1351, 0.1353. The two heaviest were used for the spectator quark. The heavy quarks were smeared with novel gauge invariant functions and the light quarks fuzzed.

We obtain the form factors from heavy-to-light three-point correlation functions, dividing by the appropriate two-point functions. We place the operator for the heavy-light pseudo-scalar at $t = 20$, rather than the midpoint of the lattice. This allows us to estimate the size of the systematic error coming from different time orderings and excited states. The tensor mixing with the vector current is included.

For $B \to \pi$ decay, results are only available for both initial ($\vec{p}$) and final ($\vec{k}$) states having zero momentum. For the D decays, a range of momentum values were used to allow comparison with pole dominance models of the form factors.

3. DECAYS OF D MESONS

The matrix elements for D to K can be parameterised in terms of two form factors [2,3],

$$\begin{aligned}\left\langle K, \vec{k}\,|V^{\mu}|\,D, \vec{p}\right\rangle &= (p + k - q\Delta_{m^2})^{\mu}\, f_+(q^2) \\ &+ \quad q^{\mu}\Delta_{m^2} f_0(q^2) \qquad (1)\end{aligned}$$

where

$$\Delta_{m^2} = \frac{m_K^2 - m_D^2}{q^2} \quad \text{and} \quad q = k - p$$

The form factors were measured for 6 different values of momentum transfer, $|p| \to |k|$: $0 \to 0$, $0 \to 1$, $1 \to 1$, $1 \to 0$, $1 \to 1_{\perp}$, $1 \to 1_{\leftarrow}$, in units of $\pi/12a$. Equivalent momentum channels are averaged over whenever possible to reduce statistical errors.

3.1. Chiral Extrapolations

The extrapolation of the form factors and meson masses to physical values of quark masses proceeds as follows. For each momentum channel the six light kappa combinations were fitted to the following functional form [2]:

$$F(\kappa_a, \kappa_p) = \alpha + \beta \left(\tfrac{1}{\kappa_p} - \tfrac{1}{\kappa_c}\right) +$$

$$\gamma\left(\frac{1}{\kappa_p}+\frac{1}{\kappa_a}-\frac{2}{\kappa_c}\right)^{\frac{1}{2}}+\delta\left(\frac{1}{\kappa_p}+\frac{1}{\kappa_a}-\frac{2}{\kappa_c}\right) \qquad (2)$$

where κ_p refers to the passive or spectator quark and κ_a to the active light quark at the W vertex. Figure 1 shows the f^+ form factor for the six light kappa combinations.

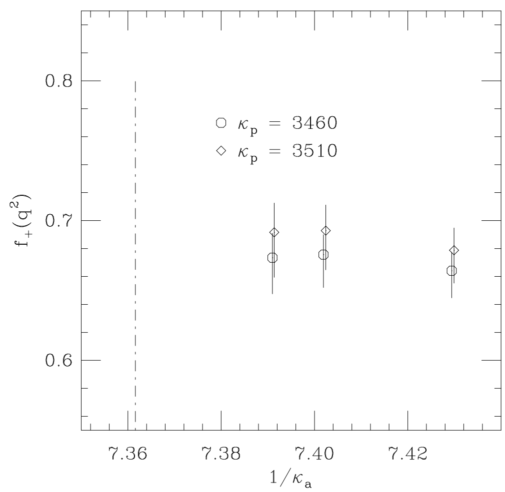

Figure 1. *Chiral behaviour of f^+ for $\vec{p}=0$, $\vec{k}=(1,0,0)$ momentum channel.*

The four-momentum transfer depends on the masses of the states. The meson masses were extrapolated using the PCAC relation and the momentum transfer calculated using the dispersion relation at the extrapolated masses. We used the values of $\kappa_{crit}=0.13844$ and $\kappa_{strange}=0.13476$.

3.2. Pole Dominance Models

Pole dominance models [3] suggest the following dependence of the form factors on q^2

$$f^+(q^2)=\frac{f^+(0)}{1-q^2/m^2_{1^-}} \quad \& \quad f^0(q^2)=\frac{f^0(0)}{1-q^2/m^2_{0^+}}$$
$$\text{with} \qquad f^+(0)=f^0(0) \qquad (3)$$

Figure 2 shows the pole mass fits to the data. The upper curves are f^+ and the lower f^0. The solid lines are a two parameter fit (fit A Table 1) for the form factor at zero momentum and the pole mass. The dashed lines (fit B table 1) are a one parameter fit to the form factors, with the pole mass fixed to the two-point correlation function value.

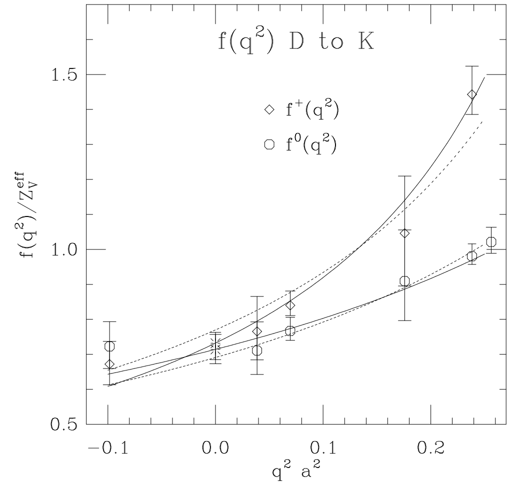

Figure 2. *Pole dominance fits to form factors. The points at $q^2=0$ are the interpolated data.*

Table 1
Pole Mass fits (masses in lattice units)

fit	$f^+(0)$	$m^{cs}_{1^-}$	$f^0(0)$	$m^{cs}_{0^+}$
A	0.732^{+31}_{-47}	0.701^{+24}_{-24}	0.714^{+43}_{-41}	0.952^{+71}_{-72}
B	0.770^{+24}_{-36}	0.754^{+2}_{-1}	0.691^{+11}_{-21}	0.884^{+12}_{-10}

The data show good agreement with pole dominance models. For fit A, there is good agreement between $f^+(0)$ and $f^0(0)$, and the masses of the poles agree with the two-point masses (the masses in fit B). For fit B, the agreement between the vector and scalar form factors is not so good but still reasonable.

All the results quoted above are for $f^n(0)/Z_V^{\rm eff}$ The renormalisation constant, $Z_V^{\rm eff}$ can be evaluated in the Alpha scheme [1], for D to K,

$$Z_V^{\rm eff}=Z_V(1+b_V m_Q a)=1.018 \qquad (4)$$

For D to π the chiral extrapolations are much harder, principally because we have to extrapolate a long way in q^2 from the data. As a result the data for D to π is not as good and the pole mass dominance models do not work as well. More work is needed on the chiral extrapolations.

4. DECAYS OF B MESONS

We have analysed the semileptonic decay $B\to\pi$ at zero recoil, i.e. for $\vec{p}=\vec{k}=0$ that is $f^0(q^2_{\rm max})$. The form factor is scaled to the B mass

by the following function, [4]

$$\theta = \left(\frac{\alpha_s(M)}{\alpha_s(M_B)} \right)^{\frac{2}{\beta_0}} \tag{5}$$

where $\beta_0 = 11$ in the quenched approximation, and $\Lambda_{\rm QCD} = 295$Mev. The form factor is renormalised by $Z_V^{\rm eff}$ and then extrapolated in $\frac{1}{M}$ with the following form,

$$Z_V^{\rm eff} f^0(q^2_{\rm max})\theta(M)\sqrt{M} = \eta + \frac{\gamma}{M} + \frac{\delta}{M^2} \tag{6}$$

4.1. Pseudoscalar Decay Constant

The mixing with pseudoscalar density is included. The chiral extrapolation is linear in the ratio f_B/f_π. Again we have the explicit mass dependence in the renormalisation constant. Defining the ratio

$$Z_A^{\rm r} = \frac{Z_A(1 + b_A a m_Q)}{Z_A(1 + b_A a m_q)} \tag{7}$$

However b_A is not known non-perturbatively, so we take a perturbative estimate [5] $b_A = 1.147$. The ratio f_B/f_π is extrapolated in $\frac{1}{M}$ in a similar fashion to f^0 except renormalised by $Z_A^{\rm r}$.

4.2. The Soft Pion Relation

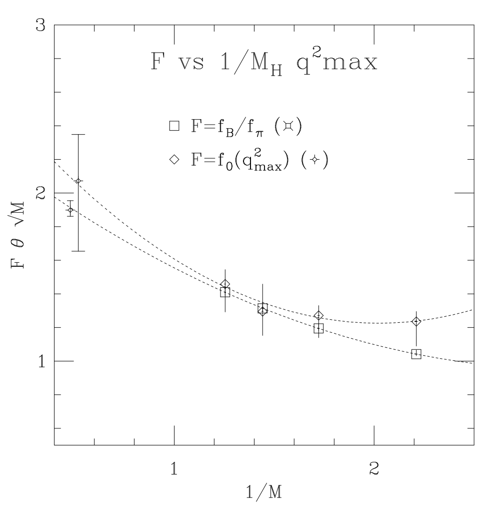

Figure 3. *Heavy mass dependence of f^0 and f_B/f_π. The end points are the extrapolated data, offset from the B mass for clarity.*

Figure 3 shows the extrapolation of f^0 overlaid with that of f_B/f_π. It is clear that the soft pion theorem is satisfied, ie

$$f^0(q^2_{\rm max}) = \frac{f_B}{f_\pi} \tag{8}$$

with

$$f^0(q^2_{\rm max}) = 1.47^{+20}_{-30} \quad \text{and} \quad \frac{f_B}{f_\pi} = 1.34^{+4}_{-3} \tag{9}$$

Two papers presented at Lattice 97 [6,7] found significant deviations from the soft pion relation. However there are several differences in the simulations. Firstly, this work includes the renormalisation constants. Secondly, this simulation is at much smaller lattice spacing. Thirdly, there are differences in the actions used. This work uses a non-perturbatively improved action, [6] uses the Tadpole improved FNAL formalism for heavy quarks and [7] uses a NRQCD action.

5. CONCLUSIONS

We find that our results for D to K decays are in agreement with pole dominance models. For B to π we find the soft pion relation satisfied.

I would like to acknowledge the support of a PPARC studentship, and EPSRC grant GR/K41663 and PPARC grant GR/L29927.

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